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Influence of the load application rate and the statistical model for brittle failure on the bending strength of extruded ceramic tiles

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Abstract

In this paper we investigate the influence of the load application rate (V) on the calculation of the bending strength (σ_B) of extruded ceramic tiles using three-points bending test as stated by ISO 10545. We also evaluate the convenience of using the average strength as the representative strength of a ceramic lot, and the goodness of the Gaussian distribution in comparison with the Weibull one to predict the behavior of the extruded ceramic material. Results show that there is a minimal influence of V on σB, although elevated velocities lead to higher results scattering. Hence, medium velocities are recommended to decrease time required for quality control during serial production. On the other hand, Gaussian and Weibull distributions differ in the prediction of the ceramic material to fracture, thus the selection of one probabilistic model should be in accordance with the design and working requirements of the each product under development.

Keywords: A. Extrusion; B. Failure analysis; C. Strength; D. Porcelain

1. Introduction

1.1. Ceramic industry and ventilated façades

Ceramic tiles have been extensively used in construction in the last years due to their aesthetics, mechanical and thermal properties. They provide an interesting and many times preferred alternative to their prime competitive products, namely, carpet, resilient flooring (vinyl, linoleum, cork and rubber), timber, laminate flooring, dimension and engineered stone [1].

Although the good results obtained by the overall market in the last decade, this industry is now undergoing deep changes in order to face the global recess of the building sector. Improvements in design and manufacturing processes as well as innovation in management and marketing have taken place. With regard to the enhancement of the performance of the tiles, both during manufacturing and along their cycle of life, the work of the researchers has been intense [2].

Extruded ceramic tiles, and particularly ventilated façades, represent a promising niche for ceramic tiles. Such systems exhibit excellent heat and acoustic insulation, in line with today's trend towards sustainable construction methods. In these systems the tiles are not directly adhered to the building walls, but are hung using mechanical clips. To develop this application it has been necessary to

Abbreviations: V, Load application rate; σ_B, Bending strength; F, Applied force at the fracture point in the tree-points bending test; S, Span between supports in the tree-points bending test; B, Width of the ceramic tile in the tree-points bending test; N, Number of specimens tested; H, Thickness of the ceramic tile at the fracture section in the tree-points bending test; σ_{Bm}, Average bending strength; y_m, Average maximum displacement; t_m, Average test duration; P, Failure probability; $\Delta\sigma_B$, Standard deviation of the bending strength; M, Weibull modulus; σ_0, Weibull characteristic stress; R_2, Quadratic error associated to the linear regression of the Weibull model; $\sigma_{Bmax}v$, Maximum percentage of variation of the bending strength with load application rate; $\sigma_{B0.5}$, Bending strength for a failure probability of $P=0.5$.

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engineer customized new systems for the installation of large porcelain tile sizes in a safe and cost-competitive way. The application of porcelain tile in ventilated façades is already a reality that may be expected to undergo great development in coming years, in view of the great efforts in research and marketing currently being made by major porcelain tile manufacturers. An example of this pursuit is the research related to bioclimatic envelopes. These envelopes, which are intended to enhance energy efficiency in buildings, represent a step forward in the concept of ventilated façades, as well as in the use of porcelain tiles in a highly sustainable environment [3,4].

1.2. Bending strength calculation

The flexural or bending strength of ceramic tiles in green, fired and polished states has been studied in the past under different perspectives [5–7]. The bending strength is a key parameter in the production and commercialization of all ceramic products, but it is extraordinarily important for ventilated façades, due to the outstanding dimensions of these tiles and the high requirements that they should meet in order to be adequate for their installation on external walls.

An appropriate value of the bending strength favors an adequate behavior during manufacturing and under working conditions. Moreover, it is mandatory to achieve certain thresholds of the bending strength to accomplish with the international standard ISO 10545–4, Ceramic tiles - Part 4: Determination of modulus of rupture and breaking strength [8]. Nevertheless, the instructions of the standard are blurry with regard to the rate of the load application up to the breaking point, possibly leading to doubtful calculations of the bending strength in practice. Considering the need for optimization that the ceramic tile companies are suffering at the present time, this kind of uncertainty is a handicap. Besides, no specific studies about the most convenient method for the calculation of the bending strength for ventilated façades haven been formally done until now.

On the other hand, ceramic is a brittle material, whose strength is a very variable property that depends on the nature of the superficial and volumetric flaws of each part. Fracture starts in general from small flaws, which are discontinuities in the microstructure and which, for simplicity, can be assumed to be small cracks distributed in the surface and/or volume. Strength then depends on the size of the largest [or critical] defect in a specimen, and this varies from specimen to specimen. Due to such variability, strength of fragile materials is not represented by a single value, but by a statistical distribution of the failure probability of a lot [9–12]. Different statistical distributions are widely used for the representation of the bending strength of advanced ceramics, but its employment on traditional ceramics is very narrow. In fact, again according to ISO 10545–4, the representative bending strength of a ceramic lot is calculated as the average of the bending strength of all samples, which involves the employment of the Gaussian [or Normal] distribution to model the mechanical behavior of the ceramic material. However, the Gaussian distribution might not be the most appropriate one in the case of the extruded ceramic tile, and other distributions such as Weibull, log normal, power law, etc. are possible candidates.

In this article we investigate the relationship between σ_B and V, the convenience of using the average strength as the representative value of the strength of an extruded ceramic lot, and the goodness of the Gaussian distribution in comparison with the Weibull one to predict the behavior of the ceramic material.

2. Materials and methods

2.1. Description of the ceramic product

A commercial lot of 80 extruded ceramic tiles of white porcelain was used in this research. These tiles were manufactured by the tile company Cerámica Mayor (Alicante, Spain) specifically for this research, so that all the variables of the manufacturing process of the lot were under control. The chemical mixture was done by means of atomic absorption.

Nominal dimensions of the tiles were 310 mm $\times$ 310 mm $\times$ 10 mm in parallelepiped shape (geometric variables L, b and h respectively).

The composition of the material was SiO_2 51.0 wt%; Al_2O_3 19.0 wt%; CaO 7.0 wt%; MgO 5.2 wt%; K_2O 3.7 wt%; TiO_2 0.7 wt%; Fe_2O_3 0.5 w.%; NaO_2 0.4 wt%; and chamotte 12.5%. This is a very common composition for the manufacturing of extruded ceramic tiles, including ventilated façades. Density was measured by immersion in mercury and fixed at the value of 2256 kg/m^3. The standard deviation of the density was 26 kg/m^3.

2.2. Experimental setup and problematic of the load application rate

A universal testing machine model MEM-101/SDC (V1.1) equipped with a three-points bending device was used to perform the mechanical strength tests as specified by ISO 10545–4. The bending tests were conducted with the parts set on two support rods separated by a span of 290 mm, applying the load through the single upper rod up to the fracture point, as shown by Fig. 1.

After fracture, the bending strength of each sample was calculated as specified by Eq. (1):

$$\sigma_B = 3FS/\left(2bh^2\right) \tag{1}$$

where σ_B is the bending strength, F is the applied force at the fracture point, S is the span between supports, b is the width and h is the thickness of the tile at the fracture section.

We considered eight different rates for the application of the load, namely, 10, 50, 100, 150, 200, 250, 300 and 500 N/s.

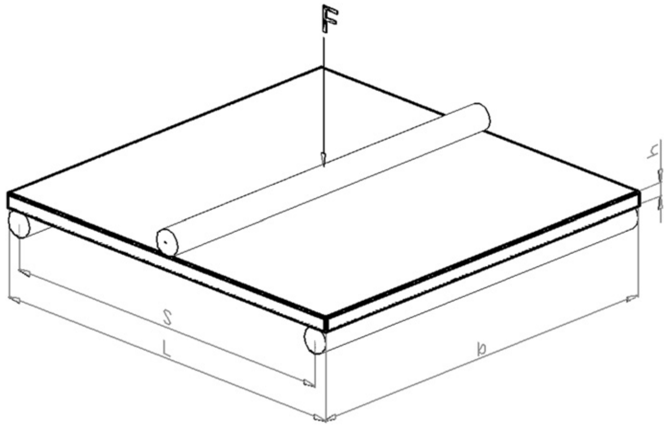

Fig. 1. Three-points bending test setup.

A set of 10 identical ceramic tiles as previously described were tested for each rate. The number of specimens (10) might seem limited. Populations between 20 and 30 samples are generally recommended by literature [13–15]. Nevertheless, previous researches focused on the bending strength of ceramic tiles have also employed sets of 10 parts [16,17]. It is out of discussion that a high number of specimens is desirable to obtain the most reliable statistical conclusions. However, the bending strength test is an expensive quality control. The preparation and subsequent destruction of the samples tested involves a cost tile companies are trying to reduce to a minimum. Therefore it is necessary to find an appropriate number of specimens for testing. According to our experience, most ceramic tile companies use sets of 10 specimens for their bending tests. Since our research tries to reproduce industrial conditions as much as possible, we have chosen this amount of parts for our work. Following the common practices of the tile industry, we pretend to develop a research work useful for the modernization of the ceramic tile sector. Moreover, a set of 10 samples exceeds the minimum prescribed by ISO 10545–4, which is 7 for the dimensions of the specimens used here.

An average bending strength was calculated for each rate as shown by Eq. 2:

$$\sigma_{\mathrm{Bm}} = \sum \sigma_{\mathrm{B}}/n \tag{2}$$

where σBm is the average bending strength, σ_{B} is the bending strength of each sample, and n is the number of specimens tested for each rate.

Average values of the maximum displacement of the tiles until fracture (y_{m}), and the duration of the test (t_{m}) were also calculated.

ISO 10545–4 states that the load F should be applied so that the stress in the part increases at a rate of (1 ± 0.2) N/mm^2 per second up to the fracture point. As no other guideline is given, apart from the Eq. (1) itself for the control of this rate, it is virtually impossible to assure this rate during the real execution of the test. In practice, each laboratory selects a typical value for this rate, which is suitable according to their expertise. Also the model of testing machine influences this choice, for not all of them allow the user to manipulate the application of the load in the same way since the control of the load application rate could be different for every model. Values around 5 mm/min have been found in recent works [7].

2.3. Analysis

For the analysis of the results, we considered both Gaussian and Weibull distributions.

2.3.1. Gaussian distribution

The Gaussian (or Normal) distribution is the most prominent statistical model in natural and social sciences, and industry. This distribution is based on the fact that most variables studied to characterize a phenomenon acquire different values depending on the particular sample under study, but all of them orbit around a central value within an interval. This central value is the average of the population for the variable under study (the most probable value, or the value which corresponds to a occurrence probability of 50%), whereas the width of the interval is measured by the standard deviation. The smaller the standard deviation, the better the Gaussian distribution represents the behavior of the variable.

The bending strength of a ceramic tile has traditionally been considered to follow a Gaussian distribution. Therefore, the probability of failure to bending strength of a single part within a lot is expressed by Eq. (3):

$$P(\sigma_{\mathrm{B}}) = 1/\Delta\sigma_{\mathrm{B}}\sqrt{(2\pi)}{*}\exp\left(\text{-}(\sigma_{\mathrm{B}}\text{-}\sigma_{\mathrm{Bm}})/2\Delta\sigma_{\mathrm{B}}{}^{2})\right) \tag{3}$$

where $\Delta\sigma$B is the standard deviation of the bending strength values for the considered population. $\Delta\sigma$B is calculated as displayed by Eq. (4):

$$\Delta\sigma_{\mathrm{B}} = \sqrt{\left(\left(\sum (\sigma_{\mathrm{B}}\text{-}\sigma_{\mathrm{Bm}})^{2/n}\right)\right)} \tag{4}$$

2.3.2. Weibull distribution

Weibull distribution has been used since its formulation in 1939 and again in 1951 [18,19] for the prediction of fatal failures in parts build with brittle materials, as ceramic tiles. With the consideration of some reviews and simplifications [20–22], the probability of failure of a single part to bending strength can be calculated as shown in Eq. (5):

$$P(\sigma_{\mathrm{B}}) = 1 - \exp\left(-\left(\sigma_{\mathrm{B}}/\sigma_0\right)^m\right) \tag{5}$$

where σ_0 is a normalizing factor, and m is a parameter that allows the scatter of the experimental mechanical strength data to be characterized.

The physical meaning of σ_0 has been largely discussed [10,21,23,24]. Here we consider that σ_0 is the stress that corresponds to a failure probability of 63%, also known as "characteristic stress". It is to mention that m, commonly known as Weibull modulus, provides the statistical basis for the treatment. A high value of m indicates a narrow range in strengths, and therefore, the considered population of parts could be referred as to be of the same material. Experience points out that m usually lies between 1 and 20 for traditional ceramics [25].

In practice, for the determination of m and $\sigma 0$, the strength data are ranked in order and assigned a probability of failure according to the formula of one estimator, usually given by Eq. (6):

$$P = i/(1+n) \tag{6}$$

where i is the order of the ith part in the mentioned rank of the strength (being $i=1$ the part showing the lowest strength), and n is the number of specimens of the population. Other estimators, such as $P=(i\text{-}0.5)/n$ have also been used, and some other are currently under research [26]. Nevertheless the influence of the small differences between estimators is not relevant for the purpose of this study, as noted in some other studies of this type.

Finally, a plot of $\ln[\ln(1/(1-P))]$ vs. $\ln(\sigma_B)$, yields a value for m, which is the slope of the linear regression. This plot also offers a value for σ_0, which can be calculated considering that the intersection of the linear regression with the vertical axis is given by –m x $\ln(\sigma_0)$. It is also common to include a linear fit of the data presented in this plot together with a calculation of R_2, which is the quadratic error of the linear fit. A high value of R_2 denotes homogeneity of the material, that could also interpreted as the existence of a single type of defects within the internal structure of the specimens.

3. Results and discussion

3.1. Adequacy of the Weibull model and influence of the load application rate on the bending strength

Results of the tests are given by Fig. 2. This figure plots P vs. σ_B for the different velocities of the load application.

Numeric values of the Gaussian parameters σ_{Bm}, $\Delta\sigma_B$; and Weibull parameters m, σ_0 and R_2 for each load application rate are summarized by Table 1. t_m, y_{max}, $\sigma_{Bmax}v$ and are also included for a better understanding of the behavior of the tile lots.

The evolution of σ_{Bm}, $\Delta\sigma_B$, m, σ_0, R_2, and y_m vs. load application rate is plotted by Fig. 3. Values have been normalized in order to plot them all together in a scale from 0 to 1.

According to the previous results, it could be stated that the Weibull distribution is a suitable model for representing the bending strength behavior of extruded ceramic tiles. This suitability is based on the high values of both, the m parameter and the quadratic error R_2 found in the linear regression of the strength data on the Weibull plot.

Considering both, the Gaussian and the Weibull models, the rate at which the load is applied during the three-points bending test has a minimal influence on the value of the bending strength, represented by σ_{Bm} and σ_0 respectively. Not a particular relationship could be found between the increment of the rate and the values of the bending strength.

Nevertheless, higher velocities lead to higher scatter, represented by the $\Delta\sigma_B$ in the Gaussian model, and the m parameter in the Weibull model. Considering that values of m close to 20 provide optimal reliability for ceramic tiles, low and medium velocities (around 50–150 N/s) offer optimal effectiveness of the test and the statistical treatment. Contrarily, very high velocities (within 300 and 500 N/s) produce disperse results, and their employment is therefore less recommendable.

Seems logical to think that low and medium velocities in the application of the load facilitate certain fluency in the facture of the samples even though ceramics are essentially brittle materials, whereas very high velocities prevent any kind of fluency and accelerate de propagation of the fracture through the flaws of the sample.

It is also to remark that all experimental values could be fit to a single straight line as exposed by Fig. 4; meaning that only one type of defect is present on the samples. Therefore, changes in the rate of the load application do not induce new families of defects on the population, although. Nevertheless

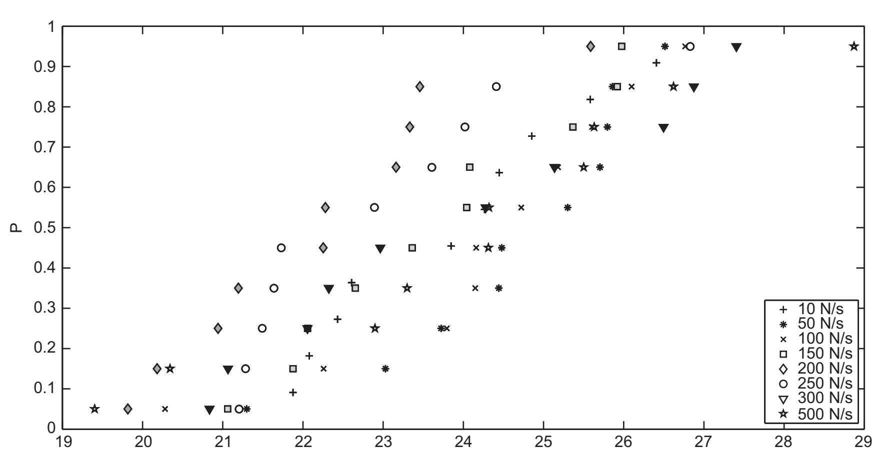

Fig. 2. P vs. σ_B plot each load application rate.

Table 1
Gaussian and Weibull distribution parameters for each load application rate.

V (N/s)	σ_{Bm} (Mpa)	$\Delta\sigma_{\mathrm{Bm}}$ (Mpa)	m	σ_0 (Mpa)	R_2	t_{m} (s)	y_{m} (mm)	$\sigma_{\mathrm{Bmax}}v$ (%)
10	23.84	1.55	14.82	24.59	0.92	160.5	1.522	17.2
50	24.62	1.58	18.18	25.31	0.98	33.2	1.599	19.7
100	24.30	1.90	14.80	25.14	0.97	16.4	1.639	24.2
150	23.64	1.74	15.94	24.40	0.93	10.6	1.584	18.9
200	22.22	1.75	14.90	22.98	0.91	7.5	1.507	22.6
250	22.91	1.82	13.71	23.77	0.75	6.2	1.552	21.0
300	23.94	2.44	11.33	25.00	0.89	5.4	1.593	24.0
500	24.12	2.82	10.02	25.31	0.97	3.2	1.529	32.8

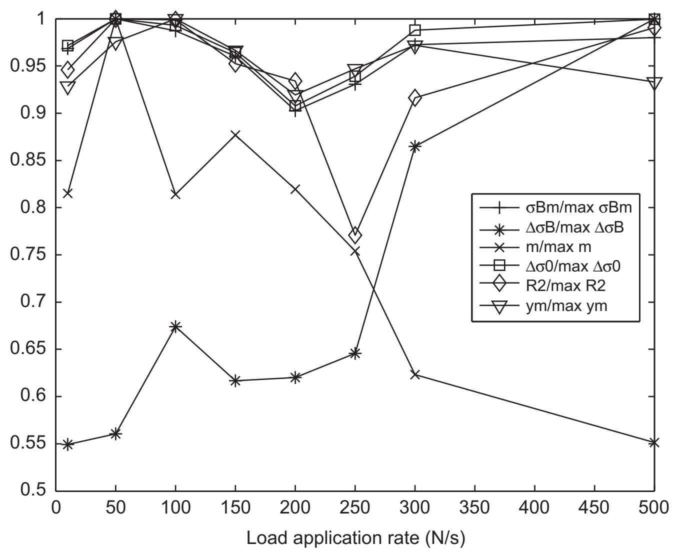

Fig. 3. Evolution of Gaussian and Weibull parameters with the load application rate.

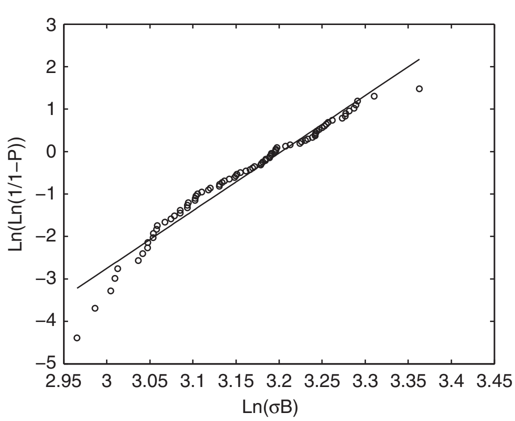

Fig. 4. Weibull plot and linear regression for the full population.

the increment of the scatter accounts for an increase of the brittle character of the material, as already said.

The evolution of y_{m} backs up the previous statements. The average maximum displacement remains quite stable for low and medium values of the load application rate, whereas it is unusually high for $V=500$ N/s.

Considering these conclusions, medium velocities could be used in order to decrease the time required for quality

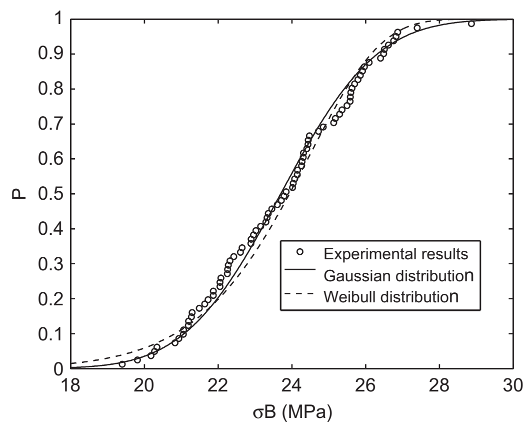

Fig. 5. Gaussian and Weibull theoretical models.

control during the serial production of ceramic tiles, without compromising the reliability of the test results.

3.2. *Calculation of the representative bending strength of a extruded ceramic lot and comparison between Gaussian and Weibull models*

Fig. 5 shows the evolution of the probability failure of all samples, and the corresponding Gaussian and Weibull theoretical models, built from the parameters given by Table 2. These parameters have been calculated from the experimental data of this study, considering the full population.

The values of the bending strength for a failure probability of 0.5 predicted by Gaussian and Weibull models are given by Table 3. These values have been calculated by solving numerically the models expressed by Eqs. (3) and (5), when $P=0.5$. As already commented, the value predicted by the Gaussian model when $P=0.5$ is equal to the average bending strength, calculated as established by Eq. (2) for the full population.

The difference between both predicted values is around 1%, being the Gaussian prediction more restrictive, this is to say, for this probability of failure, the working bending strength is smaller using the Gaussian prediction than using the Weibull prediction. Nevertheless, it could be observed that the restrictiveness of the models is not linear. In fact, the Gaussian model is more restrictive for central probabilities of failure (approximately between $P=0.2$ and

Table 2
Gaussian and Weibull theoretical distribution parameters for the full population under study.

Statistical model	Parameter	Value
Gaussian theoretical distribution	σ_{Bm} (Mpa)	23.70
	$\Delta\sigma_B$ (Mpa)	2.04
Weibull theoretical distribution	m	13.57
	σ_0 (Mpa)	24.59

Table 3
Gaussian and Weibull predictions of bending strength for $P=0.5$ for the full population under study.

Statistical model	σ_B (0.5 Mpa)
Gaussian theoretical distribution	23.70
Weibull theoretical distribution	23.94

$P=0.8$), whereas out of the central interval, the Weibull model is more restrictive. Maximal discrepancies between both models are found around $P=0.05$, $P=0.5$ and $P=0.95$. This later aspect is quite significant, since these probabilities of failure represent the most interesting cases from the point of view of the design of brittle products. Therefore, the employment of a particular statistical model for the design of an extruded ceramic tile is meaningful, and should not be taken lightly. Depending on the design and production objectives, and the quality and safety requirements for the ceramic product, the most appropriate model should be chosen. Typical extruded ceramics are finely represented by their bending strength for a probability failure of 0.5. In this case, the Gaussian model is more conservative. However, new extruded ceramics like ventilated façades are to accomplish more and more demanding requirements and so a lower failure probability could be asked. Then, the Weibull model is more conservative.

4. Conclusions

The most relevant conclusions of this work are summarized in the following paragraphs:

- The rate at which the load is applied during the three-points bending test has a minimal influence on the value of the bending strength of extruded ceramic tiles. Changes in the rate of the load application do not induce new families of defects on the samples either. Nevertheless, higher velocities cause the increment of the results scattering, accounting for an increase of the brittle character of the material. Therefore medium velocities (around 50–150 N/s) offer optimal effectiveness of the test and the statistical treatment. Considering this conclusion, test machines with a wide range of load application velocities could be used, and medium velocities could be recommended in order to decrease the time required for quality control during serial production.

- Computational simulation of laboratory tests and manufacturing processes is a essential tool of the modern industry, when it comes to the development new products with maximal productivity. The employment of such a tool is very common for advanced ceramics. However, simulations are barely known in the porcelain ceramic tile industry, and even rarer in the case of extruded materials. The simulation of the four or three-points bending test (depending of the kind of ceramic product) has become popular in the last years. These simulations have mainly used static analysis, this is to say, the applied load had a fix value, independent from time, around the typical rupture force of the material under study. Then, the stress and deformation results were gathered. Considering that V has a minimal influence on σ_B, it is justified the employment of static analysis in the computational simulation of the three-points bending test for extruded ceramic tiles.

- Both Gaussian and Weibull statistical distributions could be used to represent the bending strength of extruded ceramic tiles. Predictions differ, thus the most appropriate distribution should be chosen depending on the design and production objectives, and the quality and safety requirements for the ceramic product under development.

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