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Cement and Concrete Research 33 (2003) 1235–1244

CEMENT AND CONCRETE RESEARCH

Effect of strength and age on the stress–strain curves of concrete specimens

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Received 22 August 2002; accepted 27 January 2003

Abstract

Many researchers have rigorously studied the nonlinear behavior of the stress–strain relationship of concrete using mathematical curves. Currently, most empirical expressions for the stress–strain relationship, however, have been focused on hardened concrete and are unable to completely represent the behavior of concrete at an early age. A broad understanding of the behavior of concrete from early age to old age is important in evaluating the durability and service life of concrete structures.

In this paper, the effect of five different strength levels and various ages from 12 h to 28 days on the compressive stress–strain curve was observed experimentally and analytically. Tests were carried out on ∅ 100 × 200 mm cylindrical specimens cured in a standard moist room at a temperature of 20 ± 3 °C. An analytical expression of the stress–strain curve with strength and age was developed using regression analyses on experimental results. For verification of the proposed model equation, the equation was compared to the experimental data and existing model equations.

The result shows that the proposed model equation is not only compatible with the experimental data but also describes satisfactorily the effect of strength and age on the stress–strain curve.

Keywords: Stress–strain curve; Compressive strength; Age (of concrete); Cylinder

1. Introduction

The stress–strain curve, representing the deformation and strength characteristics, is an important material characteristic of concrete. Since the early 1900s, many researchers have tried to model the stress–strain curve. However, due to various influencing factors and different experimental conditions, the proposed curves differ.

Most model equations used presently have been developed for old-age concrete, and the appropriate equation for early age concrete is not clearly defined. However, a good knowledge of the stress–strain relationship at early ages plays an important role in the determination of time for the removal of shoring and in the calculation of thermal stresses due to the hydration heat of cement and shrinkage stresses that occur during hardening. A comprehensive understanding of the behavior of concrete members at an early age is necessary not only for the design and construction of the concrete structures but also for the evaluation of durability and service life.

The research presented in this paper examines the effect of strength levels and ages on the stress–strain relationship, considering five different strength levels and ages between 12 h and 28 days. Carreira and Chu's model equation [1] was selected as the basic equation. The stress–strain relationship at different strengths and ages was suggested based on Carreira and Chu's model equation [1], where the ascending and descending branches are considered separately.

To evaluate the adequacy of the proposed model equation, it was analyzed and compared with experimental results obtained from this study: Khan et al.'s research result [2], Carreira and Chu's model equation [1], Ahmad and Shah's model equation [3], and Kim and Lee's model equation [4]. It was determined that the stress–strain curve

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doi:10.1016/S0008-8846(03)00044-9

suggested in this study predicts the experimental data better than the other model equations, especially in the descending branch.

2. Test procedure and analysis of test results

2.1. Test procedure

In this test, a ∅ 100 × 200 mm cylindrical specimen, which is usually employed in the compressive strength test, was used. The cement used in the experiments is ordinary portland cement (ASTM Type I), and river sand was used as the fine aggregate. The crushed granite gravel passing the 19-mm sieve was used as the coarse aggregate. Mixture proportions of concrete are given in Table 1. A superplasticizer (super-20), which meets ASTM C 494 [5] requirements for Type F admixture, was also used to obtain good workability. The compressive strengths at 28 days were 15, 20, 30, 50, and 70 MPa, respectively. After casting, all specimens were subjected to moist curing. The cylinders were demolded at 10 h of age. After removal of the mold, they were stored in a standard moist room at a temperature of 20 ± 3 °C until the moment of testing.

Because it is not easy to grind the end part of the specimens at early ages, before 1 day, the capping method, using ultra rapid hardening cement, was used. The axial compressive load was applied using a universal testing machine (UTM, Closed-Loop Servo-Hydraulic Testing Machine) with a capacity of 2500 kN. For measuring the load, load cells with a capacity of 220 and 2200 kN were used for low- and high-strength levels, respectively. A compressometer, composed of two linear variable displacement transducers (LVDTs) with 10.0-mm ranges and supporting assembly, were mounted diametrically opposed on the lower platen to measure the platen-to-platen displacement of the specimen. The experiment was performed using a displacement control method of 0.003 mm/s velocity. The rate was applied continuously and without shock. Loads and displacements were measured continuously using load cells and LVDTs until the specimen failed.

Table 1
Concrete mixture proportions

w/c (%)	s/a (%)	Unit weight (kg/m^3)					Ad[a] (%)
		W	C	S	G[b]	S.P.[c]	
89	49.5	185	208	925	950	68	–
69	40.0	185	268	717	1085	–	0.15
54	42.0	185	342	727	1012	–	0.3
39	45.4	175	449	799	970	–	0.8
30	42.6	170	567	714	970	–	1.1

[a] Superplasticizer (high-range, water-reducing admixture), ratio of cement weight.

[b] Maximum aggregate size of 19 mm.

[c] Stone powder.

2.2. Analysis of test results

Water/cement ratios (w/c) of 0.89, 0.69, 0.54, 0.39, and 0.30 were used. For each w/c ratio, the stress–strain curves were obtained at 12 h, 18 h, 1 day, 2 days, 3 days, 7 days, 14 days, and 28 days. The results are shown in Fig. 1. In this figure, the test results reported are the averaged results from three identical ∅ 100 × 200 mm cylinders at each age in the series.

Characteristics of the stress–strain curves obtained from experiments at each age with the five different strength levels are as follows:

1. At early ages after casting, very low elastic modulus, low strength, and very high strain corresponding to the maximum stress were found. With increasing age, however, elastic modulus and strength increased rapidly and the strain corresponding to the maximum stress decreased rapidly.
2. For each mixture, at approximately 14 days after casting, the stress–strain curve changed rapidly, and after 14 days the curve was similar to that of 28 days.
3. Strain corresponding to the maximum stress of the stress–strain curve was largest at early ages. However, the strain decreased rapidly with developing strength until, at high strength, the strain increased again slowly.
4. At an age of 28 days, the strain corresponding to the maximum stress was larger with higher strength level.
5. With smaller w/c value, the increase of elastic modulus with age was faster.
6. At more than 14 days at w/c = 0.39 and more than 2 days at w/c = 0.30, the decreasing branch of the stress–strain curve could not be obtained due to the rapid brittle failure of concrete and the inability of the closed-loop system to compensate.

3. Development of the stress–strain curve

3.1. General

The actual stress distribution in the compression zone of concrete members is extremely difficult to measure and to adequately model. The shape of the stress–strain curve changes with various factors. Thus, the shape is very complex because the factors do not act independently. It is not easy to represent the influencing factors as constants.

However, an equation representing the stress–strain curve of concrete should meet the following conditions:

1. The equation should compare favorably with experimental data from carefully conducted experiments.
2. Ascending and descending branches should be shown.
3. The equation should be based on physically significant parameters that can be experimentally determined. At the

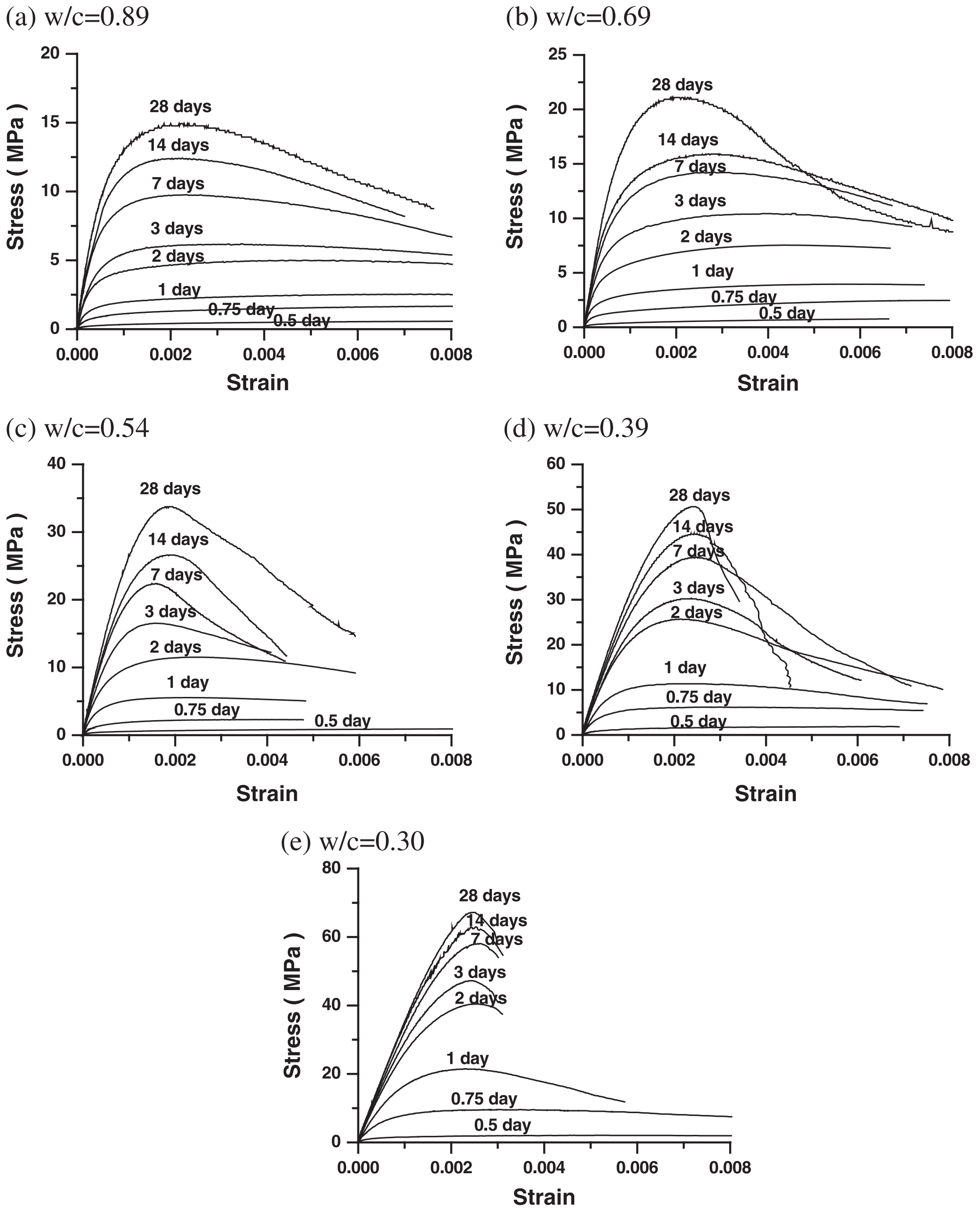


Fig. 1. Compressive stress–strain curves at eight different ages with five different w/c.

point of origin, $d(f)/d\varepsilon = E_{it}$, where f is the concrete stress, ε is the concrete strain, and E_{it} is the initial tangent modulus.

4. At the point of maximum stress, $d(f)/d\varepsilon = 0$.

The most common parameters with physical significance used to define the stress–strain curve include the following:

1. f_c' is the maximum stress, usually considered as the compressive strength of concrete and determined in accordance with ASTM C 39 [6], "Standard Test Method for Compressive Strength of Cylindrical Specimens."
2. ε_c' is the strain corresponding to the maximum stress f_c'.
3. E_c is the modulus of elasticity.
4. E_{ci} is the slope at the origin or initial tangent modulus.
5. ε_{cf} is the ultimate strain or strain at which failure is defined.

3.2. Modeling of the stress–strain curve

3.2.1. Selection of the basic model equation

As discussed in the foregoing, a form of model equation has been suggested by many researchers [1–4,7–9]. In the present study, the shape suggested by Carreira and Chu [1] was introduced as the basic model equation. Because the equation is simple, the shape of the stress–strain curve at early ages is well represented by the change of β value (see Fig. 2), and it has good correlation with the experimental results of other researchers (e.g., Hognestad et al. [10], Kaar et al. [11], and Wang et al. [12]). It can be concluded that the equation is a verified equation. Therefore, the suggested model equation is based on Carreira and Chu's model equation [1], and the effect of strength and age on β value is additionally considered.

Carreira and Chu's model equation [1] is as follows:

$$\frac{f_c}{f'_c} = \frac{\beta(\varepsilon_c/\varepsilon'_c)}{\beta - 1 + (\varepsilon_c/\varepsilon'_c)^{\beta}} \qquad (\text{for } \beta \geq 1.0 \text{ and } \varepsilon_c \leq \varepsilon_{cf}) \tag{1}$$

As given by Eq. (1), β as a function of initial tangent modulus E_{ci} can be estimated by:

$$\beta = \frac{1}{1 - \dfrac{f'_c}{\varepsilon'_c E_{ci}}} \tag{2}$$

Additionally, β can also be represented as a function of the modulus of elasticity E_c [1] as follows:

$$\left(0.4\frac{f'_c}{E_c\varepsilon'_c}\right)^{\beta} - \beta\left(\frac{f'_c}{E_c\varepsilon'_c} - 1\right) - 1 = 0 \tag{3}$$

The ratio of the secant modulus of elasticity at the apex E_o ($=f'_c/\varepsilon'_c$, modulus of elasticity, which connects the origin and the apex of the stress–strain curve) to the initial tangent modulus increases with developing strength, and the value β in Eq. (2) also increases. Accordingly, the slope of the stress–strain curve increases and the pattern is similar to Fig. 2.

Table 2
Material properties with w/c of concrete

w/c (%)	Age (days)	E_o	E_c	E_o/E_c	$\beta_{m,a}$ (fitted)	$\beta_{m,d}$ (fitted)
89	1	320	5513	0.06	1.05	1.40
	2	1011	11675	0.09	1.06	1.42
	3	1767	11740	0.15	1.11	1.48
	7	4193	17334	0.24	1.21	1.60
	14	5528	21982	0.25	1.28	1.70
	28	6573	23152	0.28	1.29	1.75
69	0.5	78	334	0.25	1.29	–
	0.75	353	2943	0.12	1.17	–
	1	647	8711	0.08	1.06	1.42
	2	1727	14450	0.12	1.11	1.47
	3	2619	17707	0.15	1.12	1.60
	7	5131	20317	0.25	1.26	1.65
	14	5621	22220	0.25	1.25	1.95
	28	9947	25329	0.39	1.54	2.24
54	0.5	88	1089	0.09	1.12	–
	0.75	520	5229	0.10	1.08	–
	1	2570	13008	0.20	1.17	1.47
	2	4513	18806	0.24	1.22	1.66
	3	10673	22995	0.46	1.70	1.85
	7	13911	26261	0.53	1.93	2.63
	14	13508	26948	0.51	1.82	3.00
	28	17324	28881	0.60	2.30	–
39	0.5	294	2237	0.13	1.12	–
	0.75	2060	11939	0.17	1.13	–
	1	4934	17629	0.28	1.28	1.75
	2	12076	25496	0.47	1.76	2.19
	3	12802	26173	0.49	1.86	3.06
	7	15774	28243	0.56	2.14	3.18
	14	18355	29970	0.61	2.49	5.19
	28	20581	31225	0.66	2.76	7.18
30	0.5	383	3620	0.11	1.10	–
	0.75	2747	13067	0.21	1.17	–
	1	9231	21327	0.43	1.67	2.46
	2	15912	28106	0.56	2.20	4.08
	3	19434	30411	0.64	2.69	5.69
	7	22092	32657	0.68	3.00	6.23
	14	24790	34619	0.72	3.52	7.52
	28	27203	35581	0.76	4.53	–

However, Carreira and Chu's model equation [1] has a defect. Eq. (2) is a representation of β value by the initial tangent modulus. Although there is a model code [13] that can mathematically obtain the initial tangent modulus, the value is not correct. It can be obtained using an experimental technique; however, this is not practical due to the difficulty of the procedure to obtain it. In Eq. (3), the mathematical equation representing β as a function of the elastic modulus

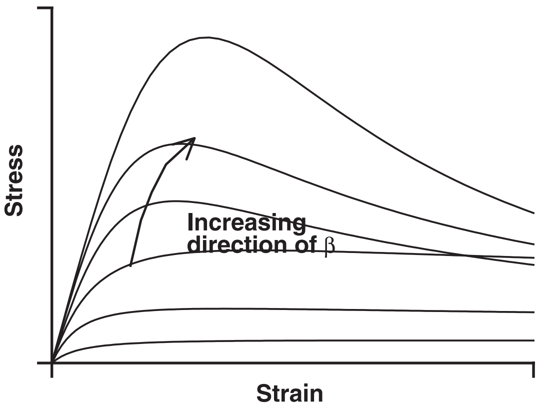


Fig. 2. Effect of β on shape of stress–strain curves.

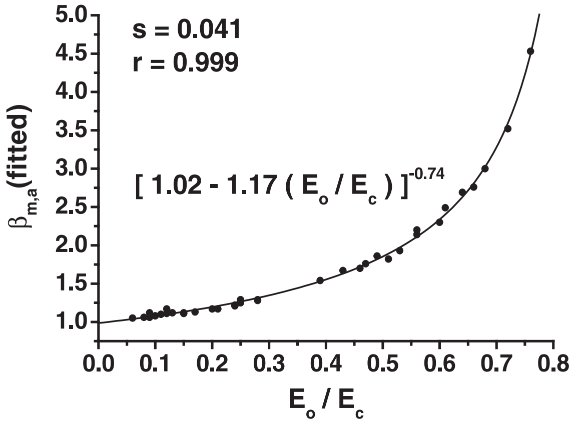


Fig. 3. Relationship between $\beta_{m,a}$ (fitted) and E_o/E_c.

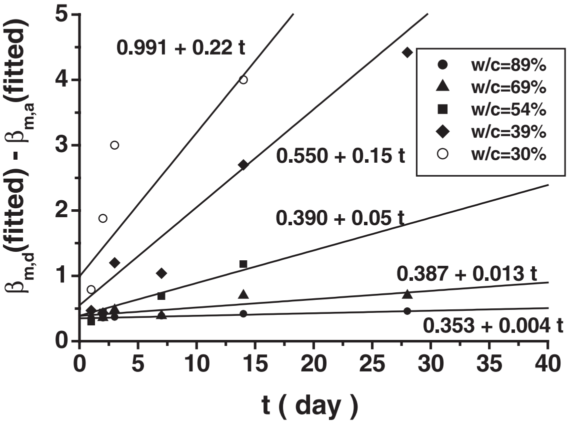


Fig. 4. Relationship between $\beta_{m,d}$ (fitted) − $\beta_{m,a}$ (fitted) and age *t*.

has an error that reduces β to a value less than 1.0 for low-strength concrete or at early ages. Therefore, it is necessary to represent the actual stress–strain curve using material properties, which can be used more easily.

3.2.2. Development of model equation

E_o/E_c has been used by many researchers [4,7–9] as the most important influencing factor in determining the stress–strain curve. In this study, it is also used as the most important factor. Its effect, including age (t) and compressive strength at 28 days (f_{28}), on the stress–strain curve was investigated. A comparison was made between the E_o/E_c effect and experimental results. The results show that there is an apparent difference in the effect of E_o/E_c on the ascending and descending branches. Thus, the ascending and descending branches were considered separately and the modified β (i.e., β_m) was obtained by the following procedure. β_m was obtained from Eq. (3) since the analytical development of this experiment is not impacted by the problem with Eq. (3). This was confirmed from Fig. 3.

3.2.2.1. Ascending branch $\beta_{m,a}$. The value of $\beta_{m,a}$ for the ascending branch is obtained from the correlation between the values of $\beta_{m,a}$ (fitted) predicted from the regression analyses of experimental results and material properties as indicated previously. To obtain the correlation, Table 2 shows E_o (MPa), E_c (MPa), E_o/E_c, and $\beta_{m,a}$ (fitted) with age at each mixture. In this table, the symbol "−" means that the descending branch is not available at an early age, the fitting result is not adequate, and/or the descending branch was not obtained in the high-strength cases. Therefore, the corresponding data were eliminated.

As can be seen in Table 2, the values of E_o and E_c increase rapidly with age, and E_o/E_c also increases rapidly since E_o increases more than E_c. In addition, E_o/E_c increases with the decrease of w/c at the same age.

In this table, E_o/E_c and $\beta_{m,a}$ (fitted) are correlated. The correlation between them is shown in Fig. 3 with age at each mixture. From Fig. 3, it is noted that between $\beta_{m,a}$ (fitted), the slope of the basic model equation Eq. (1), and E_o/E_c, the following correlation is present.

$$\beta_{m,a}\ (\text{fitted}) = [1.02 - 1.17(E_o/E_c)]^{-0.74} \tag{4}$$

Fig. 3 show the rapid increases of $\beta_{m,a}$ (fitted), departing from a linear relationship at a value of E_o/E_c between 0.4 and 0.5. Therefore, it can be defined that the slope of the stress–strain curve increases rapidly after developing a certain strength level and passing a certain age.

3.2.2.2. Descending branch $\beta_{m,d}$. The stress–strain curve represented by Eq. (1) has a defect, which underpredicts the decline in the stress–strain curve. Even for low-strength concrete, brittleness increases due to the hardening of concrete with age. In this case, the slope of the descending branch increases more rapidly than high-strength concrete, which has the same strength and younger age. From the test results, it was found that the slope of the descending branch drops rapidly after developing brittleness with hardening to a certain level. Therefore, to establish the value of $\beta_{m,d}$ in the descending branch, the increase in the slope will be added to the value of $\beta_{m,a}$ in the ascending branch.

To determine the slope adjustment, the difference between $\beta_{m,d}$ (fitted) estimated from the experimental results for the

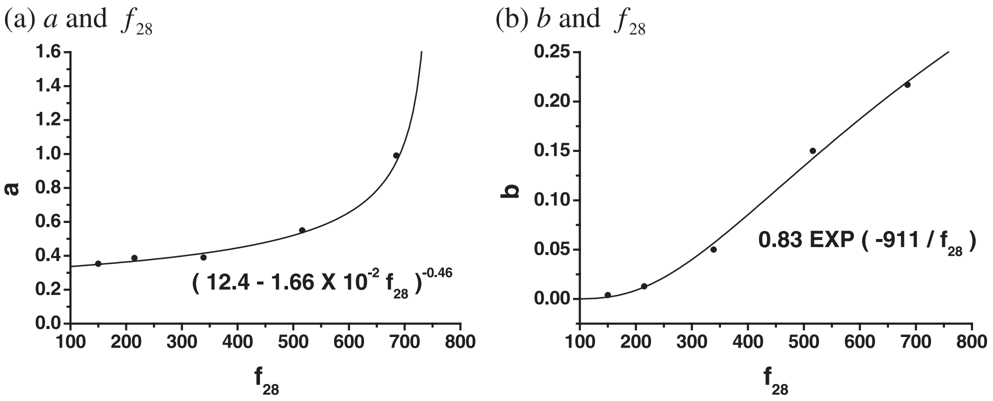

(a) *a* and f_{28} (b) *b* and f_{28}

Fig. 5. Relationship between parameters (*a*, *b*) and compressive strength at 28 days (f_{28}).

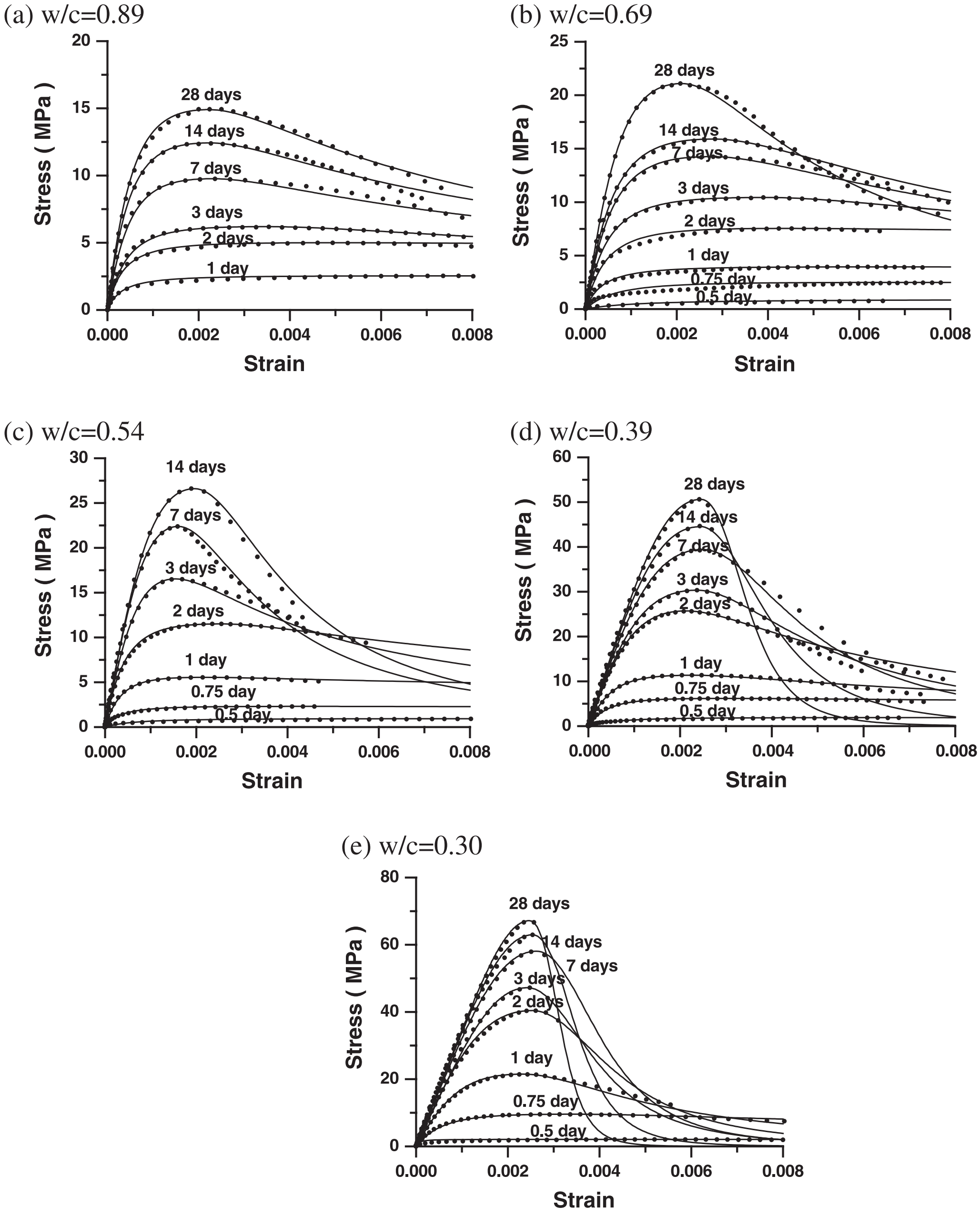


Fig. 6. Comparison of proposed model equation with experimental results.

descending branch and $\beta_{m,a}$ (fitted) obtained before, $\beta_{m,d}$ (fitted) $-$ $\beta_{m,a}$ (fitted) was evaluated with age *t*, and the results are shown in Fig. 4. From this figure, it is found that the slope adjustment with increasing age, $\beta_{m,d}$ (fitted) $-$ $\beta_{m,a}$, (fitted), increases linearly. At each mixture, the slope increased with increasing age and the slope is steeper with smaller w/c. If the slope at each mixture is represented as a linear equation, $a+bt$, the relationship between a, b, and compressive strength of concrete at 28 days, f_{28}, is shown in Fig. 5.

3.2.2.3. Suggestion of model equation. From the formal analytical results, the following equation representing the stress–strain curve as a function of age *t* (day), compressive strength at 28 days f_{28}, E_o, and elastic modulus E_c, according to ASTM C 469 [14], was proposed.

$$\frac{f_c}{f'_c} = \frac{\beta_m(\varepsilon_c/\varepsilon'_c)}{\beta_m - 1 + (\varepsilon_c/\varepsilon'_c)^{\beta_m}}$$

$$\beta_m = \beta_{m,a}\ (\text{fitted}) = [1.02{-}1.17(E_o/E_c)]^{-0.74} \quad (\varepsilon_c \le \varepsilon'_c)$$

$$\beta_m = \beta_{m,d}\ (\text{fitted}) = \beta_{m,a}(\text{fitted}) + (a + b \cdot t) \quad (\varepsilon_c \ge \varepsilon'_c)$$

$$a = (12.4 - 1.66 \times 10^{-2} f_{28})^{-0.46}$$

$$b = 0.83\exp(-911/f_{28}) \tag{5}$$

4. Verification of the proposed model equation

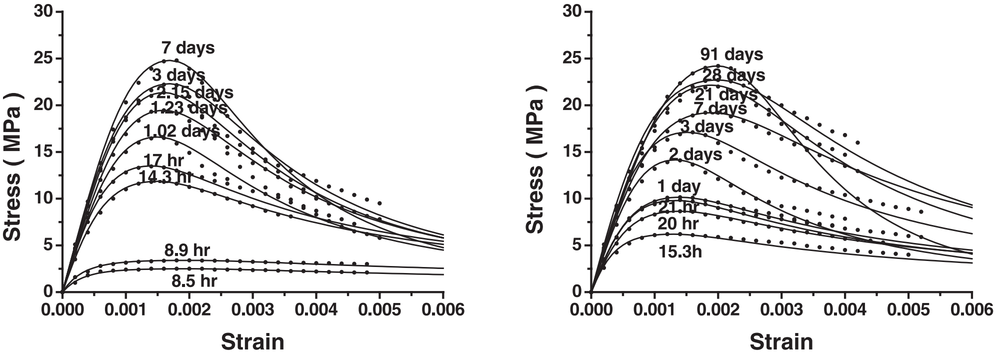
4.1. Comparison of the proposed model equation and the experimental results of this study

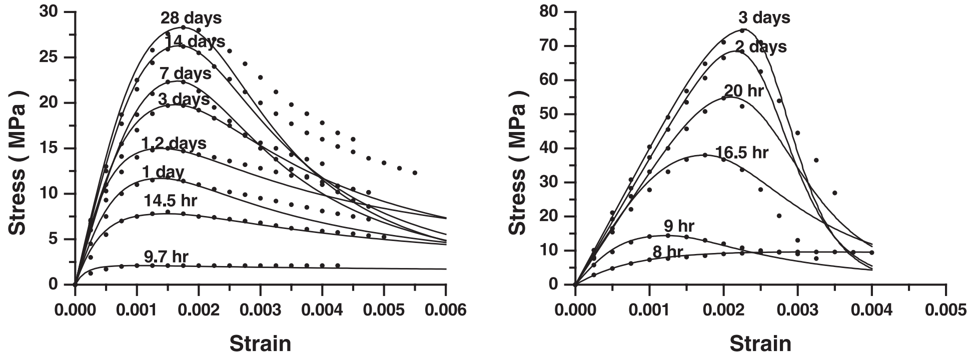
To ascertain the accuracy of the proposed model equation, its usefulness and reliability should be determined by comparing it with reliable experimental results. According to the existing research results [15], there is a difference between the stress–strain curves since the shape and size of specimens, strain rate, specimen versus machine stiffness, displacement control method, etc. are different. Based on the existing studies, the ascending branch of the model equations does not show much difference; however, in the descending branch the difference is apparent. These results were expected because the displacement control method of the experiments is different. In this experiment, to obtain a more stable descending branch, the rate of loading was 0.003 mm/s.

The modeling work comparison was performed based on the stress–strain curve for each mixture, and the reliability of the model was verified by comparing and analyzing the difference between the suggested model equation and the experimental results of this study.

The proposed model equation and the experimental results were compared based on the age *t* (day), compressive strength at 28 days f_{28}, and E_o/E_c. After evaluating how well the proposed model equation represents the stress–strain curve, the results are shown in Fig. 6.

Fig. 6 shows that the stress–strain curve, with age in the ascending branch, has a good correlation with the


(a) Temperature-matched curing ($f_c{}'$ = 30 MPa) (b) Air-dried curing ($f_c{}'$ = 30 MPa)


(c) Sealed curing ($f_c{}'$ = 30 MPa) (d) Temperature-matched curing ($f_c{}'$ = 70 MPa)

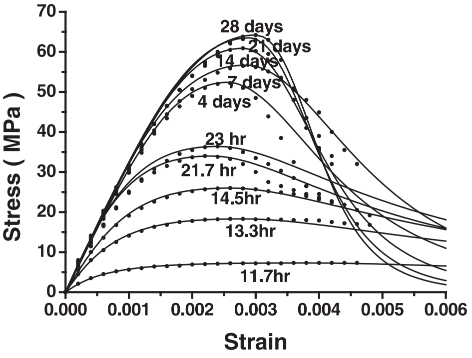

(e) Air-dried curing ($f_c{}'$ = 70 MPa)

Fig. 7. Comparison of proposed model equation with Khan et al.'s [2] experimental results.

experimental data. However, there is an error of minor magnitude in the descending branch. Based on the fracture properties of concrete, cracks do not occur in a stable and regular manner beyond the maximum stress point. As a result, the descending branch of the stress–strain curve is expected to be unstable. In the present study, the experimental results of specimens semihardened at early ages showed a good agreement with the curve. However, in the specimen hardened with increasing age, the descending branch of the experimental results showed a minor difference.

4.2. Comparison of the proposed model equation and Khan et al.'s experimental results

Most experimental research results on the stress–strain curves of concrete performed to date are for an age of 28 days. It is not easy to find the experimental results that range from early age to old age.

In the study carried out by Khan et al. [2], the stress–strain curves after 6 h were obtained on the following three curing conditions: (1) temperature-matched curing (2) sealed curing, and (3) air-dried curing. These con-

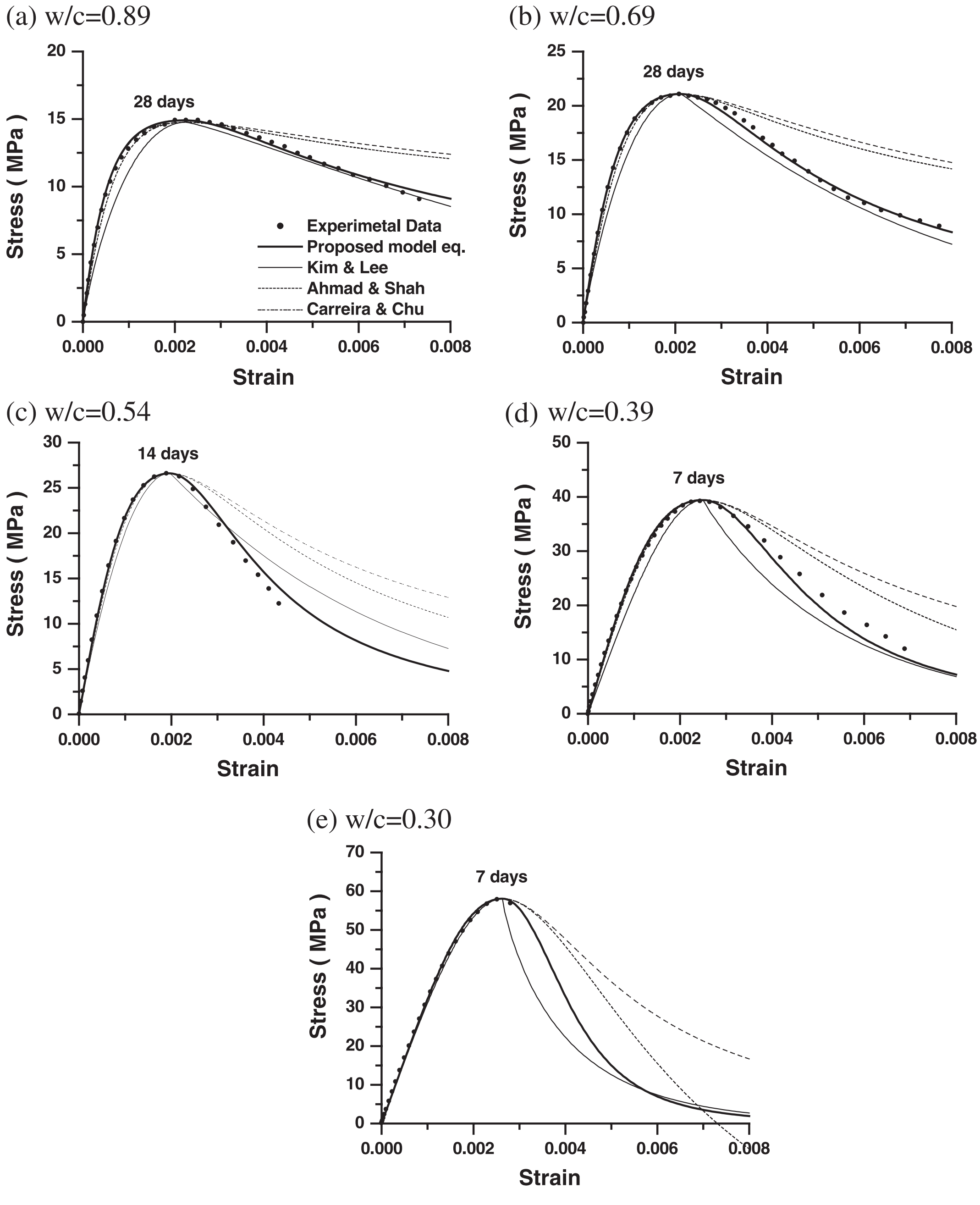


Fig. 8. Comparison of proposed and existing model equations with experimental results.

ditions modeled the curing effects at the center of mass concrete, close to the surface, and at the surface of a structural member, respectively. According to their results, there is also a difference in the shape of curves with curing condition and extent of hardening. In addition, it was found that the curve shape of this study was similar to that of Case 1.

Fig. 7 show comparisons between the proposed model equation and Khan et al.'s data [2] with strength in the selected curing condition. From this figure, it can be seen that the proposed model represents the experimental results well except the descending branches of curves corresponding to the 14 and 28 days of Fig. 7(c). Accordingly, the proposed model equation based on the experimental results of this study is well matched with the experimental data obtained from Ref. [2]. Therefore, it can be concluded that the proposed equation is reasonably general and accurate.

At the ages of 14 and 28 days of Fig. 7(c), however, a difference between the experimental data and the proposed model is present. The reasons are as follows: (1) at the ages, the β values are not increased consistently with increasing age and the trend is different from Fig. 2 and typical others' experimental data, and (2) there is a difference in the displacement control method and in the curing condition adopted. Namely, in the study of Khan et al. [2], all the specimens were tested in a strain-controlled mode (recommended by ASTM C 39 [6]) to obtain postpeak responses and investigated under three curing conditions different from this study.

4.3. Comparison of proposed and existing model equations and experimental results

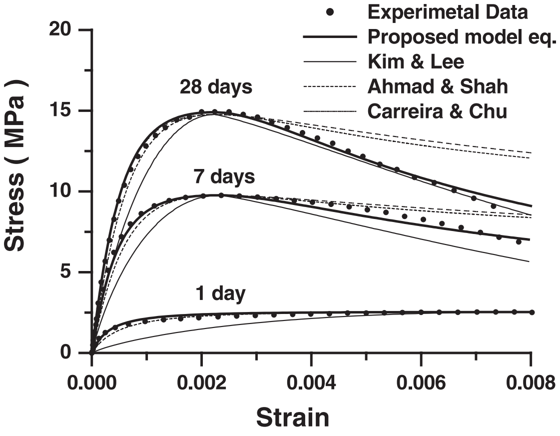
The model equations suggested by previous experimental work represent concrete in a hardened condition. To verify the proposed model equation, a comparison has been carried out on hardened specimens (refer to Fig. 8). Model equations used in comparison were as follows: (1) Carreira and Chu's model equation [1], (2) Ahmad and Shah's model equation [3], and (3) Kim and Lee's model equation [4].

As shown in Fig. 8, existing model equations generally show a trend underestimating the slope of the descending branch. It is assumed that a difference in the slope of the descending branch of each model equation occurs due to the difference of displacement control method adopted in each test. It is also observed that the descending branch of the proposed model equation generally falls between Carreira and Chu's model equation [1], Ahmad and Shah's model equation [3], and Kim and Lee's model equation [4]. In addition, the difference of slopes in the descending branch of each model equation increases more in higher strength concrete.

In low-strength concrete, the hardening is slow since the speed of hydration of cement is slow. Evaluation of the stress–strain curve with age of low- and high-strength concrete was performed on the proposed model equation, experimental results obtained in this paper, and existing model equations; the results are shown in Fig. 9.

As shown in Fig. 9(a), the decreasing slope at increased strain is more pronounced at greater age. This effect is less for low-strength concrete compared to high-strength concrete. The difference between the proposed model equation and the existing model equations also increases with increased strain. In high-strength concrete (Fig. 9(b)), the difference between these model equations with age is increased compared with low-strength concrete; and at the age of 28 days the descending branch could not be obtained by experiments due to the increased brittleness. In this case, the properties were estimated using the experimental data of 1, 2, and 7 days.

(a) w/c=0.89



(b) w/c=0.39

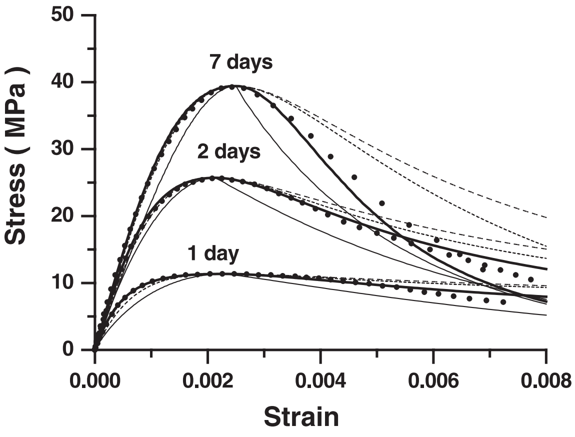


Fig. 9. Comparison of stress–strain curves with age.

5. Conclusions

In this study, the effect of concrete strength and age on the stress–strain curve of concrete was investigated and the results can be summarized as follows:

1. Using the experimental results and the existing model equations, a more reasonable model equation to consider the effect of concrete strength and age was proposed.
2. The model equation suggested in this study accurately predicts the ascending branch of the stress–strain curve, and it also predicts the descending branch well within a minimum range of deviations.
3. In the hardened concrete, the ascending branch of the model equation proposed in this study and the existing model equations showed a similar pattern. However, a considerable difference was found in the descending branch. This is because the proposed model equation can consider the slope of the descending branch while most existing model equations cannot reflect the influence of increasing slope with age in the descending branch.

Notation

a,b	coefficients of linear equation $a+bt$
E_c	modululs of elasticity
E_o	secant modulus of elasticity ($=f_c'/\varepsilon_c'$)
E_{ci}	slope at the original or initial tangent modulus
f	concrete stress
f_{28}	compressive strength of concrete at 28 days
f_c'	maximum stress, compressive strength of concrete
t	age
β	material parameter that depends on the shape of the stress–strain curve
$\beta_{m,a}$	modified material parameter at the ascending branch
$\beta_{m,d}$	modified material parameter at the descending branch
ε	concrete strain
ε_c'	strain corresponding with the maximum stress f_c'
ε_{cf}	ultimate strain or strain at which failure is defined
s/a	sand/(sand+gravel)
S	sand
G	gravel
Ad	admixture

Acknowledgements

The authors would like to thank the Korea Institute of Science and Technology Evaluation and Planning (KISTEP) for the financial support of the National Research Laboratory (NRL).

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