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Modeling for prediction of restrained shrinkage effect in concrete repair

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Abstract

A general model of autogenous shrinkage caused by chemical reaction (chemical shrinkage) is developed by means of Arrhenius' law and a degree of chemical reaction. Models of tensile creep and relaxation modulus are built based on a viscoelastic, three-element model. Tests of free shrinkage and tensile creep were carried out to determine some coefficients in the models. Two-dimensional FEM analysis based on the models and other constitutions can predict the development of tensile strength and cracking. Three groups of patch-repaired beams were designed for analysis and testing. The prediction from the analysis shows agreement with the test results. The cracking mechanism after repair is discussed.

Keywords: Free shrinkage; Tensile creep; Concrete repair; Restrained shrinkage; Cracking

1. Introduction

Cracking caused by restrained shrinkage in patch repair work influences the durability of concrete structures. The cracking is related to many factors; the key factor is the shrinkage of repair material at early age. Tensile strength, modulus of elasticity, tensile creep of repair material at early age, and the structural characteristics also govern the effect of restrained shrinkage [1,2]. For further investigation on the cracking mechanism and modified prediction of the cracking based on the work of Yuan and Marsszeky [1], testing and modeling about free shrinkage and tension creep of repair material at early age are carried out. Two-dimensional analyses based on the models and experiments on patch-repaired concrete beam are carried out as well.

2. Shrinkage model of repaired material at early age

The shrinkage of cement-based material comprises chemical shrinkage, thermal strain and drying shrinkage [3,4]. The shrinkage of epoxy-based material comprises chemical shrinkage and thermal strain, and no drying shrinkage. The chemical shrinkage during the hardening process is mainly discussed in the paper.

The hardening process is a complex chemical reaction process in cement-based and epoxy-based materials. Although different kinds of materials have a different chemical reaction process, a general model will be developed.

2.1. Chemical shrinkage model

During the material hardening, it is assumed that

(1) ambient temperature is kept constant;
(2) mass is constant during hardening;
(3) shrinkage in transverse direction is neglected.

According to the above assumptions, Eq. (1) can describe the chemical shrinkage.

$$\varepsilon_{\mathrm{cm}}(t) = \frac{1}{L_0}\int_0^t \frac{\mathrm{d}L}{\mathrm{d}t}\mathrm{d}t = \frac{1}{AL_0}\int_0^t \frac{\mathrm{d}V}{\mathrm{d}t}\mathrm{d}t \qquad (1)$$

Here, $\varepsilon_{\mathrm{cm}}(t)$ is the chemical shrinkage, L_0 the original length of test sample, A the area of test sample section, V_0 the original volume and t the chemical reaction time (age of repair material).

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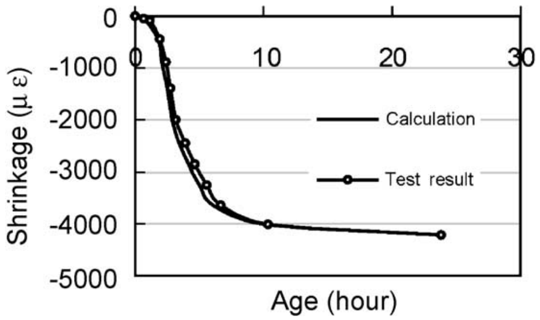

Fig. 1. Development of chemical shrinkage in UP resin mortar.

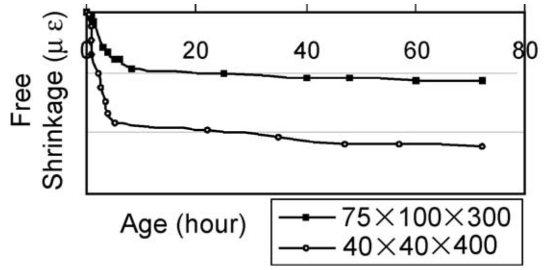

Fig. 2. Size effect on chemical shrinkage in UP resin mortar.

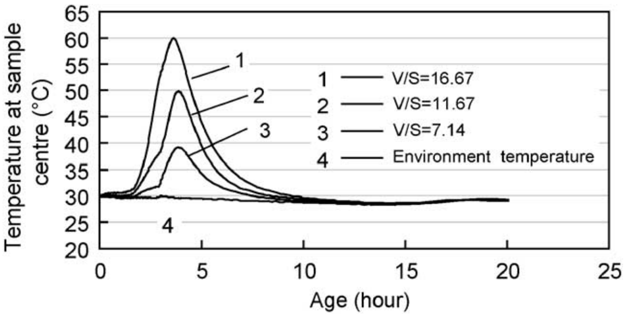

Fig. 3. Variety of internal temperatures in the different-size samples.

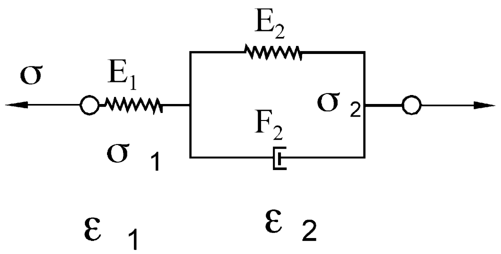

Fig. 4. Physical model for tensile creep.

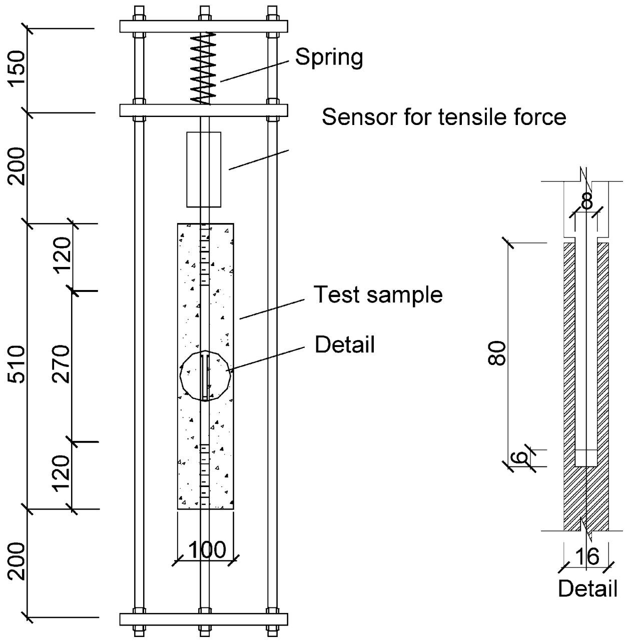

Fig. 5. Set-up for tensile creep.

A degree of chemical reaction $\alpha(t)$ is defined as

$$(\alpha)_T = \left(\frac{\varepsilon_{as}(t)}{\varepsilon_{as}(\infty)}\right)_T \qquad (2)$$

Eq. (3) expresses the increment of the volume change due to chemical reaction in accordance with the definition of the degree of chemical reaction.

$$\Delta V = \int_0^t \frac{dV}{dt}\,dt = V\int_0^t \left(\frac{d\alpha}{dt}\right)dt \qquad (3)$$

A relationship between chemical reaction rate and reaction temperature is indicated by Arrhenius' law [3].

$$\left(\frac{d\alpha}{dt}\right)_T = K_4 e^{\frac{-E}{R_g T}} \qquad (4)$$

Here, E is the activation energy, R_g the universal gas constant (8.314 J/mol K), T the reaction temperature and K_4 the constant of reaction rate.

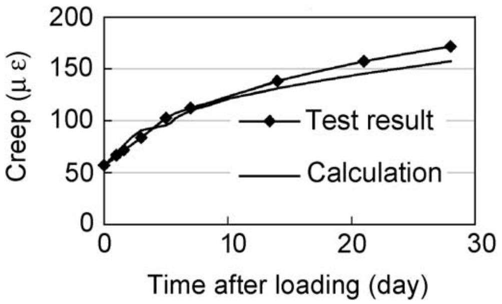

Fig. 6. Development of tensile creep in SBR-modified cement mortar.

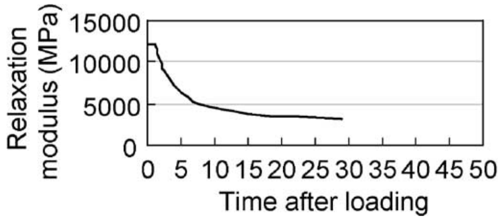

Fig. 7. Development of relaxation modulus in SBR mortar.

Eq. (3) can be rewritten by means of Arrhenius' law and replaced by Eq. (5).

$$\begin{aligned}\varepsilon_{\mathrm{cm}}(t) &= \frac{1}{AL_0}\int_0^t \frac{\mathrm{d}V}{\mathrm{d}t}\mathrm{d}t = \frac{VK_4}{AL_0}\int_0^{t_{\mathrm{eq}}(t)} \exp\left(\frac{-E(t)}{R_{\mathrm{g}}T_{\mathrm{ref}}}\right)\mathrm{d}t \\ &= K_4\sum_0^{t_{\mathrm{eq}}(t)} \exp\left(\frac{-E(t)}{R_{\mathrm{g}}T_{\mathrm{ref}}}\right)\Delta t \qquad (5)\end{aligned}$$

$$t_{\mathrm{eq}}(t) = \sum_1^n \exp\left[-\frac{E(t)}{R_{\mathrm{g}}}\left(\frac{1}{T_{\mathrm{i}}} - \frac{1}{T_{\mathrm{ref}}}\right)\right]\Delta t$$

Here, T_{i} is the average temperature in Δt time interval.

Table 1
List of samples for analysis

Beam No.	Repair material	Depth of patch, d (mm)	Reinforcement in tension area
A-1	Resin mortar	75	2Φ10 stirrup Φ4@75
A-2	Resin mortar	50	2Φ10, Φ4@75
A-3	Resin mortar	75	–
B-1	SBR modified	75	2Φ10, Φ4@75
B-2	SBR modified	50	2Φ10, Φ4@75
B-3	SBR modified	75	–
C-1	Cement mortar	75	2Φ10, Φ4@75
C-2	Cement mortar	50	2Φ10, Φ4@75
C-3	Cement mortar	75	–

2.2. Testing on free shrinkage

Three kinds of repair materials including unsaturated polyester (UP) resin mortar, SBR-modified cement mortar and normal cement mortar are chosen for the free shrinkage testing. The mixture composition (weight ratio) of repair materials is as follows:

UP resin mortar: UP resin:hardener:accelerator:filler = 100:4:1.5:300;
SBR-modified mortar: silica fume:SBR mixture:cement: sand:water = 1:0.5:9:14:4;
Normal cement mortar: cement:sand:water = 1:2.5:0.46.

The sample size is 40 × 40 × 400 mm. The temperature of the testing environment is 20 ± 2 °C and humidity 75 ± 10%. The sample was sealed by vinyl sheet to prevent evaporation. The thin vinyl sheet was sealed by grease and sealing effect was tested by weighing a special sample. It is assumed that mass is constant during hardening and the drying shrinkage is neglected in the test.

The sample can freely shrink in the test setup. The temperature caused by chemical reaction in the sample was measured by a thermocouple, which was positioned in the test sample. The data about the temperature were collected automatically every 5 min.

The mathematical model about chemical shrinkage for various repair materials can be established by means of Eq.

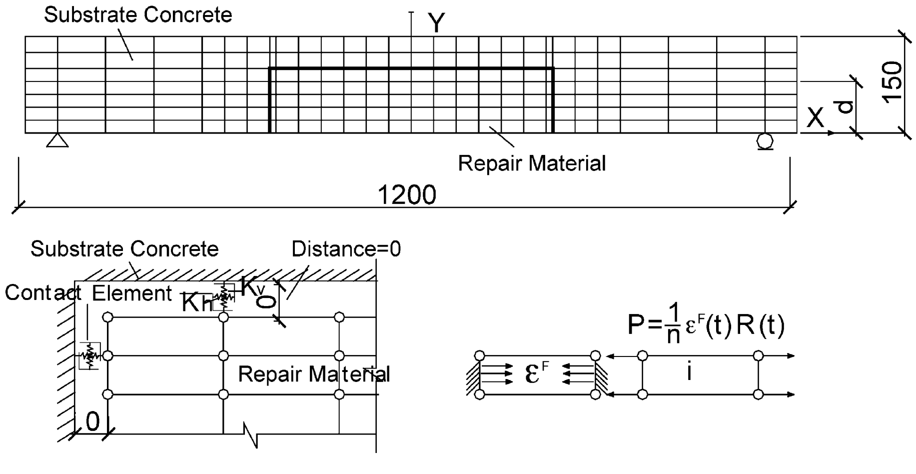

Fig. 8. FEM model for time-dependent analysis.

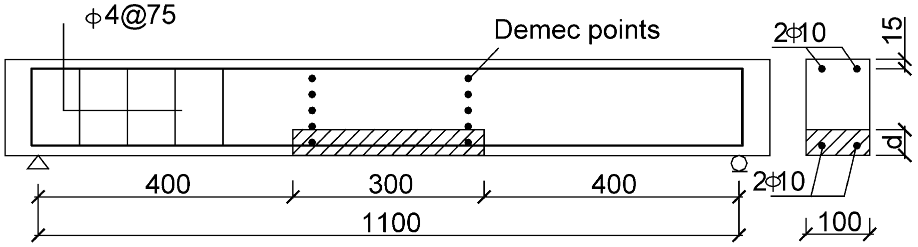

Fig. 9. Beam sample after repair (A-1).

(5) and its test results. A comparison of the model prediction with the experimental data is presented in Fig. 1.

2.3. Size effect

Obvious size effect on chemical shrinkage in UP resin mortar is shown in Fig. 2. The temperature in the sample is higher than the temperature in the environment due to chemical reaction. Higher internal temperature is found in the bigger-sized sample (V/S). The obvious thermal difference between surface and internal temperature causes thermal expansion, which offsets the shrinkage partly. Fig. 3 shows a variety of internal temperatures in the different-size samples. The size effect on chemical shrinkage in cement-based material is distinctly lower than in UP-resin-based material.

3. Tension creep model of repair material at early age

3.1. Physical model

A viscoelastic model [5], shown in Fig. 4, is introduced to develop a tensile creep model.

The constitution equations based on the viscoelastic model are as follows,

$$\sigma_1 = E_1\varepsilon_1 \tag{6}$$

$$\sigma_2 = E_2\varepsilon_2 + F_2\dot{\varepsilon}_2 \tag{7}$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{8}$$

Here, E is the elastic modulus and F the viscous damping coefficient.

The differential Eq. (6) can be established using Laplace transformation,

$$\sigma + p_1\dot{\sigma} = q_0\varepsilon + q_1\dot{\varepsilon} \tag{9}$$

Here

$$p_1 = \frac{F_2}{E_1 + E_2} \quad q_1 = \frac{E_1F_2}{E_1 + E_2} \quad q_0 = \frac{E_1E_2}{E_1 + E_2}$$

The solution of differential equation based on initial conditions is given by:

$$\varepsilon(t,\tau) = \frac{\sigma_0}{q_0}\left(1 - \left(1 - \frac{p_1q_0}{q_1}\right)e^{-\frac{q_0}{q_1}(t-\tau)}\right) \tag{10}$$

Here, τ is material age at loading, t is the material age and σ_0 is the initial stress.

Stress relaxation due to creep can be expressed by means of the differential equation and its initial condition.

$$\begin{aligned}\sigma(t-\tau) &= q_0\varepsilon_0\left(1 - e^{\frac{-(t-\tau)}{p_1}}\right) + \frac{q_1}{p_1}\varepsilon_0 e^{\frac{-(t-\tau)}{p_1}} \\ &= \varepsilon_0 R(t-\tau)\end{aligned} \tag{11}$$

$$R(t-\tau) = q_0 + (q_1/p_1 - q_0)e^{-(t-\tau)/p_1}$$

Here, $R(t-\tau)$ is the relaxation modulus and ε_0 the initial strain.

Table 2
Comparison between analysis and test results

Test sample		A-1	A-2	A-3	B-1	B-2	B-3	C-1	C-2	C-3
Cracking time	Test	22[a] h	21 h	24 h	–	5 days	–	4.75 days	4.25 days	5.25 days
	Analysis	25 h	24 h	27 h	–	–	–	5 days	4.5 days	5.5 days
Cracking position	Test	1	1	1	–	2	–	2	2	3
	Analysis	1	1	1	–	–	–	2	2	3

Cracking position 1: in concrete near transverse interface; position 2: on the transverse interface; position 3: on the repair material near transverse interface.

[a] Cracking time in test is not accurate time; the error is ±6 h.

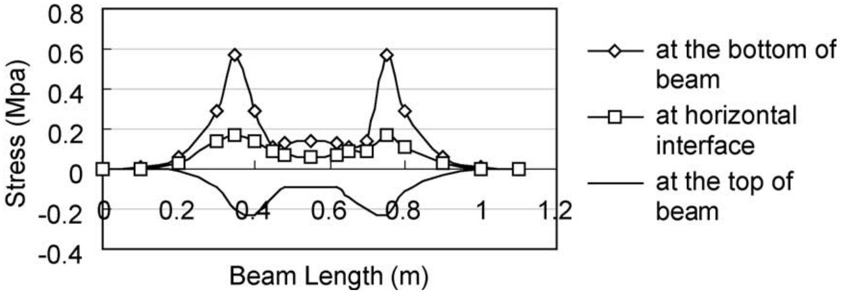

Fig. 10. Longitudinal distribution of normal stress.

3.2. Testing on tension creep

The tests of tensile creep and free shrinkage are carried out under the same environment and same-dimension sample. The dimensions of the sample for the tests are 60 × 60 × 508 mm.

The initial loading age for UP resin mortar is at 24 h, and for cement-based materials at 3 days. The loading level is 20% of the tensile strength of the material at 28 days age. The initial loading is 2.67, 0.45 and 0.34 Mpa for UP resin mortar, SBR-modified cement mortar and normal cement mortar, respectively. Constant and axial loading can be ensured in the test setup shown in Fig. 5. Creep strain was measured automatically at intervals of 3 h.

The parameters in the model of creep and relaxation modulus are dependent on the test results (Figs. 6 and 7).

$$q_0 = \sigma_0 / \varepsilon(\infty, \tau) \tag{12}$$

$$q_1 = q_0 \frac{1}{\ln\left[\frac{\varepsilon(\infty,\tau)-\varepsilon(\tau,\tau)}{\varepsilon(\infty,\tau)-\varepsilon(t,\tau)}\right]} \tag{13}$$

$$p_1 = q_1 \varepsilon(\tau, \tau)/\sigma_0 = q_1/E_1 \tag{14}$$

Here σ_0 is the initial stress; $\varepsilon(\infty,\tau)$ the ultimate creep and $\varepsilon(t,\tau)$ the creep at time t.

4. Prediction of restrained shrinkage effect in patch repair

4.1. Beam model for two-dimension analysis

A beam model for patch repair was built for FEM analysis. The elements of concrete, repair material and bond element in the beam model are shown in Fig. 8. A linear element is used to represent steel element. A bond element between steel bar and concrete (or repair material) is defined as a no-dimension element, and its bond stress–slip model is established by test results.

The following constitutive models are used in the FEM analysis:

(1) stress–strain model of concrete and steel bar,
(2) free shrinkage model including chemical shrinkage model, thermal strain model and drying shrinkage model,
(3) tensile creep model and its relaxation modulus model, and
(4) bond strength model under tension and shear.

Time-dependent analyses are carried out; no external loading effect is considered.

4.2. Beam sample for analysis and test

Three groups of beam samples, shown in Table 1, are designed for analysis and testing. The geometric dimension is shown in Fig. 9. The age of the beam sample is 16 months.

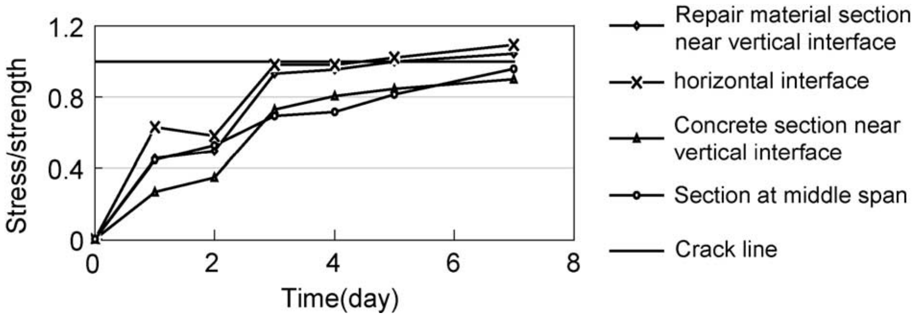

Fig. 11. Development stress in test beam repaired by normal cement mortar.

The shrinkage of concrete in the beam sample is neglected. The patch repair work and measurements are under the same environment as the free shrinkage and the creep test.

4.3. Effect of restrained shrinkage in patch repair [6]

4.3.1. Cracking prediction

The prediction of the restrained shrinkage effect from analysis results shows agreement with test results. The results are shown in Table 2.

4.3.2. Longitudinal distribution of normal stress

The longitudinal distribution of normal stress of the beam repaired by UP resin mortar is shown in Fig. 10. Stress concentration can be found in the region of the interface because of different moduli of elasticity between concrete and repair material.

4.3.3. Development of the tensile stress in patch repair

The development of the tensile stress in relation to its strength and the moment and position that cracking occurs is shown in Fig. 11. The cracking position is found where tensile stress is beyond the ultimate stress concerned.

5. Conclusions

(1) A general model of autogenous shrinkage caused by chemical reaction (chemical shrinkage) can be established by means of a degree of chemical reaction $\alpha(t)$ and Arrhenius' law. Size effect on chemical shrinkage in UP-resin-based repair material should be considered.
(2) A general relaxation modulus model can be expressed in accordance with a viscoelastic model. The thermal and shrinkage effect should be deducted from test results.
(3) The parameters in the model of chemical shrinkage and tensile creep are dependent on the material behaviour and its test results.
(4) The prediction of restrained shrinkage effect is in agreement with test results of patch repair. The FEM analyses in two dimensions can show the development of tensile stress and cracking caused by restrained shrinkage.
(5) Tensile stress concentration can be found in the region of the transverse interface due to the difference in elastic moduli between concrete and repair material. The position of the crack can be decided depending on where the tensile stress exceeds the tensile strength first.
(6) Cracking is mainly dependent on the free shrinkage of repair material. The tensile creep can release the tensile stress caused by restrained shrinkage.

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